Teaching Material on

Elementary Mathematics

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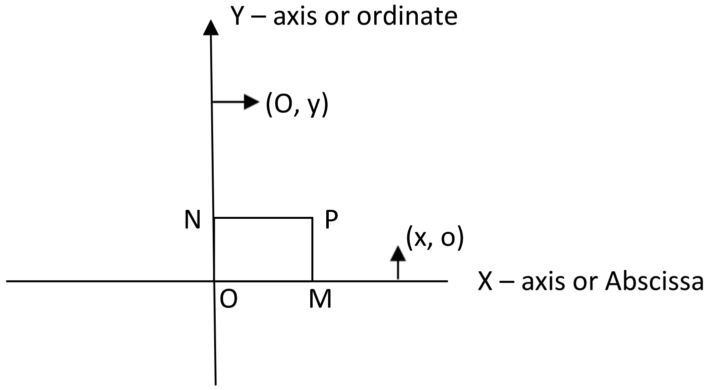
Co-ordinate Geometry



Distance PN or OM is known as x – Co-ordinate

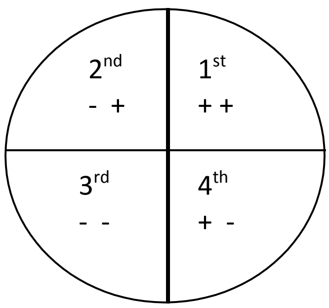
PM or ON is known as y co-ordinate or ordinate. (x,y) – co-ordinates.

Y – Co-ordinate is zero, the points which are on x-axis. (x,0)

X – Co-ordinate is zero, the points which are on y- axis (0,y)

Both Co-ordinates of origin is (0,0)

Quadrants

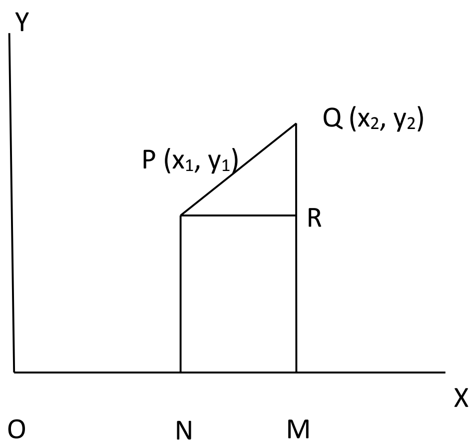


Distance Formula:

Let P and Q be two points with co-ordinates (x_1, y_1) and (x_2, y_2) respectively. Draw perpendicular from P to PN and Q to QM on axis and P to PR on QM.

\ $ON = x_1$
$OM = x_2$
$PN = y_1$ □
$QM = y_2$

$PR = NM = OM – ON = x_2 – x_1$
$QR = QM – RM = QM- PN = y_2 – y_1$



Now in rt. Angled □ PQR
$PQ^2 = PR^2 + QR^2$
$= (x_2 – x_1)^2 + (y_2 – y_1)^2$
\ $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: 1. Prove that the points (2a, 4a), (2a, 6a) and (2a +□3a, 5a) are the vertices of an equilateral triangle whose side is 2a.
$AB^2 = (2a – 2a)^2 + (6a . 4a)^2 = 4a^2$

$BC^2 = (2a + □3a – 2a)^2 + (5A - 6)^2 = 3a^2 + a^2 = 4a^2$

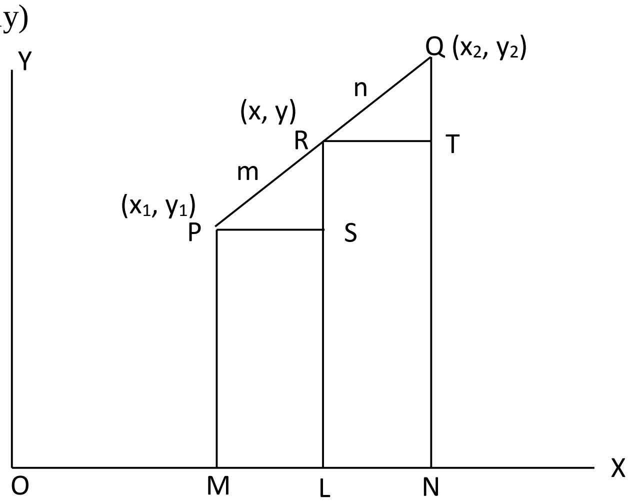
$AC^2 = (2a + □3a – 2a)^2 + (5a – 4a)^2 = 4a^2$

\ AB = BC = AC = 2a

Hence triangle is equilateral. Proved

Section formula:

To find the co-ordinates of the point which divides the line joining the points P (x_1, y_1) and Q (x_2, y_2) in the ratio m:n(internally)



Let P (x_1, y_1) and Q (x_2, y_2) the two points R be the points which co-ordinates are (x, y) on PQ such that which divides $\frac{PR}{RQ} = \frac{m}{n}$

Draw perpendicular from the point P,R,Q to PM, RL and QN on x-axis respectively and also draw perpendicular from P to PS on RL and from R to RT on QN.

\ OM = x_1 OL = x PM = y_1

ON = x_2 RL = y QN = y_2

\ PS = ML = OL – OM = $x – x_1$

RT = LN = ON – OL = $x_2 – x$

RS = RL – SL = RL – PM = $y – y_1$

and QT = QN – TN = QN – RL = $y_2 - y$

Now □s PRS and QRT are similar

\ $\frac{PS}{RT} = \frac{PR}{RQ}$

or, $\frac{S – S_1}{S_2 – S} = \frac{m}{n}$

or, $n(x – x_1) = m(x_2 – x)$

or, $nx – nx_1 = mx_2 – mx$

or, $nx + mx = nx_1 + mx_2$

or, $x(m+n) = nx_1 + mx_2$

\ $x = \frac{n S_1 + m S_2}{m + n}$

Similarly from □s PRS and QRT

$\frac{RS}{QT} = \frac{PR}{RQ}$

Or, $\frac{y - y_1}{y_2 - y} = \frac{m}{n}$

Or, $m(y_2 - y) = n(y - y_1)$

Or, $my_2 - my = ny - ny_1$

Or, $my + ny = my_2 + ny_1$

Or, $y = \frac{n y_1 + my_2}{m+n}$

Hence co-ordinates of R (x, y) = $\left(\frac{n S_1 + mS_2}{m+n}, \frac{n y_1 + my_2}{m+n}\right) \rightarrow (1)$

If m = n

Then co-ordinates of R (x,y) = $\left(\frac{m S_1 + mS_2}{2m}; \frac{m y_1 + my_2}{2m}\right)$

Hence co-ordinates of the middle point = $\left(\frac{S_1 + S_2}{2}; \frac{y_1 + y_2}{2}\right)$

General Co-ordinates

If we put $\frac{m}{n} = l$ where l is constant

$m = nl$

Then co-ordinates of R = $\frac{n S1 + nl S2}{nl + n}, \frac{n y1 + nl y2}{nl + n} = \frac{S_1 + l S_2}{l + 1}, \frac{y_1 + l y_2}{l + 1}$

$X = \frac{S_1 + l S_2}{l + 1}, y = \frac{y_1 + l y_2}{l + 1}$

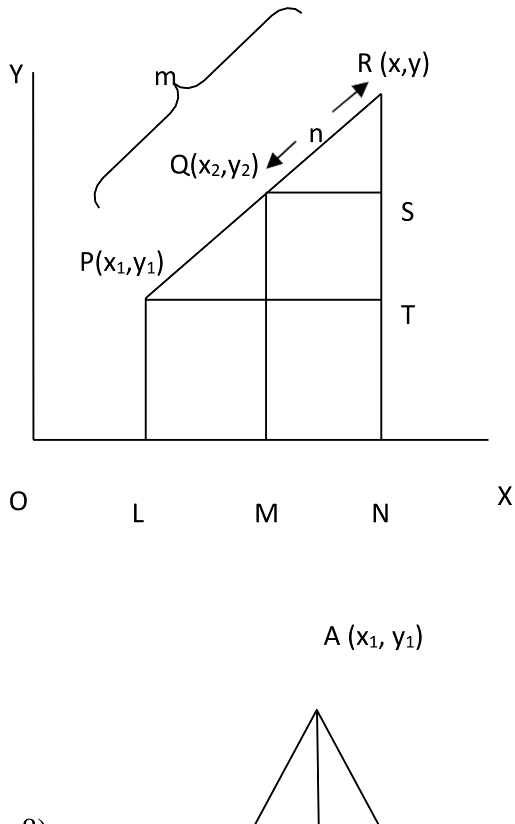
Externally

In □^{s} PRS & QRS

$\frac{PT}{QS} = \frac{PR}{QR}$

$\frac{S + S_1}{S - S_2} = \frac{m}{n}$, $nx - nx_1 = mx - mx_2$, $(m - n) = mx_2 - nx_1$

\ $x = \frac{m S_2 - n S_1}{m - n}$ and $y = \frac{m y_2 - n y_1}{m - n}$



Co-ordinates of the centroid of a triangle.

Let D be the middle point of BC

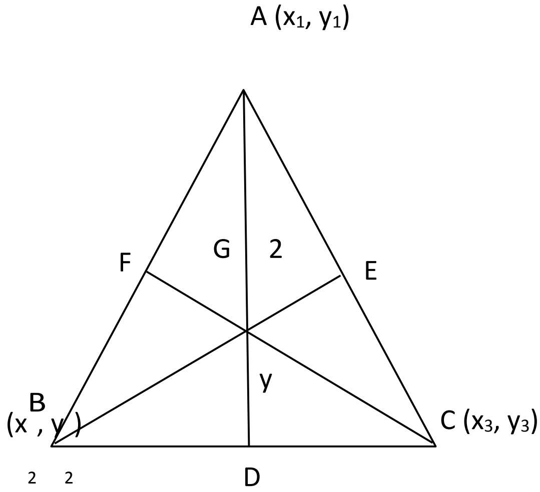
\ Co-ordinates of D = $\frac{S2 + S3}{2}, \frac{y2 + y3}{2}$

Let G be the controid of a triangle whose co-ordinate is (α, β)

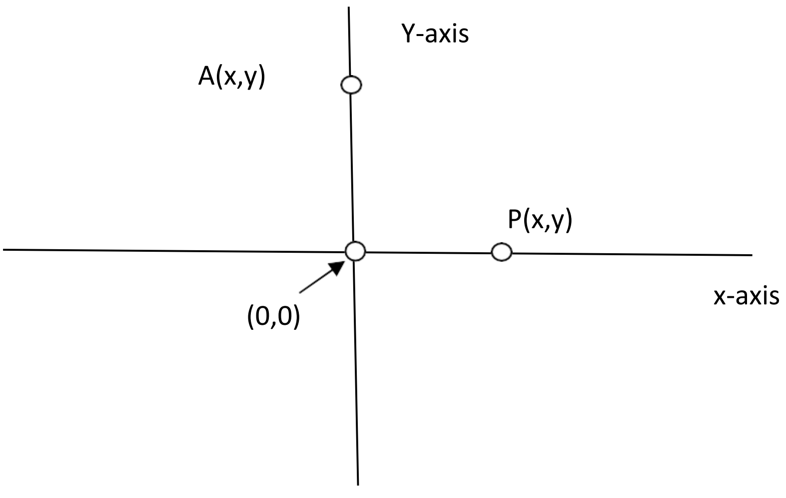
G divides the AD in the ratio of 2:1

$\alpha = \frac{1 S_1 + 2 . \frac{x2 + x3}{2}}{2+1}$, $\beta = \frac{1 y_1 + 2 . \frac{y2 + y3}{2}}{3}$

\ $\left(\frac{S_1 + S_2 + S_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ Required co-ordinate.



Equation of co-ordinate axis (x-axis & y-axis)



Equation of x- axis

Let p(x, y) be any point on the x-axis then

Y=0

Therefore equation of x axis is y=0

Equation of y-axis

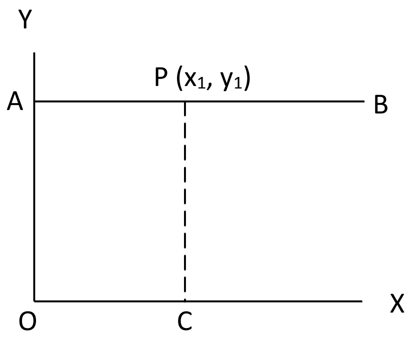
Let us take any point A(x,y) on the y-axis then

X=0

Therefore equation of y-axis is x=0

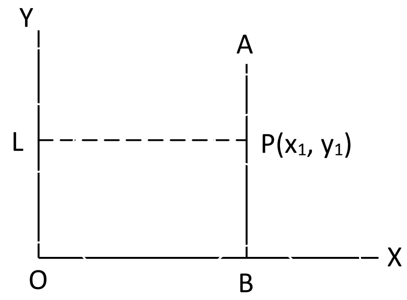
Equation of straight line parallel to x-axis is

y =d, OA = d



Equation of the straight line parallel to the y-axis is

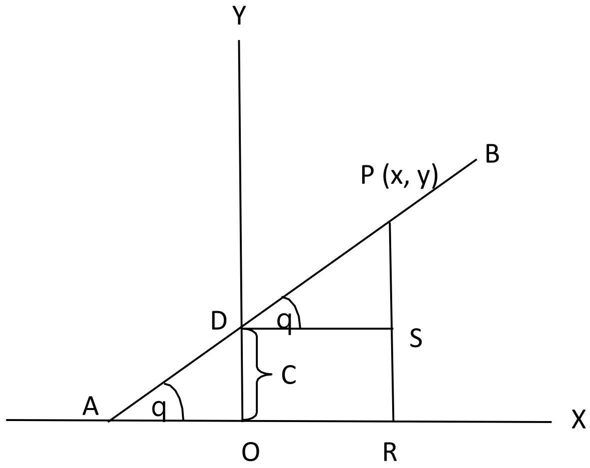
x = c, OB = c



Equation of straight line in slope intercept form:

Show that the equation of a straight line, which cuts the y- axis at a distance c from the origin and makes an angle q with the positive direction of the x-axis is y = mx + c where m = tanq

Proof:



Let AB be a straight line which cuts the y axis at a distance C from the origin O i.e. OD = c and makes an angle q with the positive direction of x – axis i.e. at point A and < BAO = q let P be the any point on the straight line whose c-ordinate be (x, y)

Draw □r frm P to R on x-axis and D to DS on PR

OD= SR = c, OR= DS = x and PR = y

PS = PR – SR = PR – OD = y – c

Since < BAO = q \ < BDS is also q

Now from right angled □ PDS

$\tan q = \frac{PS}{DS} = \frac{y-c}{s}$ or, $m = \frac{y-c}{s}$ or, y = mx + c

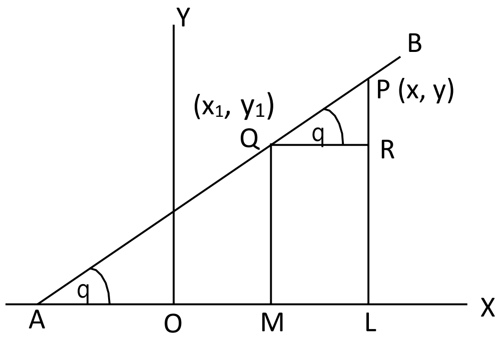
This is the required equation of st. Line AB. Proved.

Equation of straight line in slope point form:

Show that the equation of a line passes through a given point and having m as its as its slope is $y-y_1 = m(x-x_1)$

Let AB be a straight line passes through a given point $Q(x_1, y_1)$ and makes an angle θ with x-axis

Such that m = tan θ



Let P be the another point on the same line whose co-ordinate is (x,y) Draw □ r from Q to QM and P to PL on x-axis and Q to QR on PL

Since < Q A O = θ \ < PQR = θ

$QR = ML = OL – OM = x – x_1$

$PR = PL – RL = PL – QM = y- y_1$

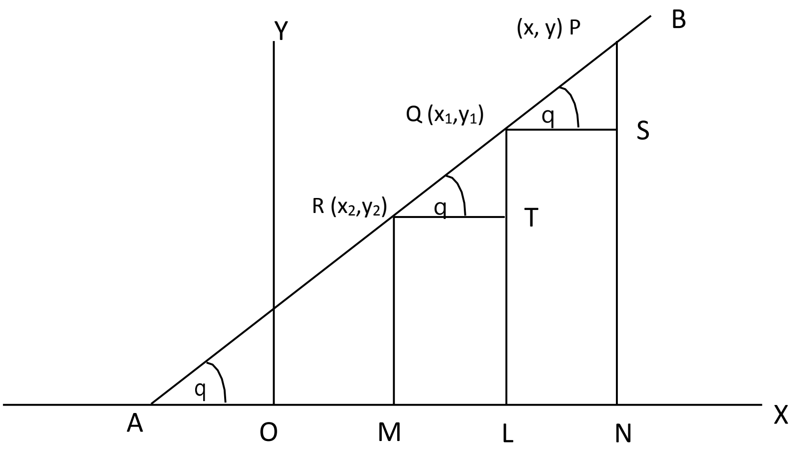
\ □ PQR, tan θ = PR/QR = $\frac{y- y1}{S- S_1}$

or, m = $\frac{y- y1}{S- S_1}$

\ $y – y_1 = m (x – x_1)$ This is the required equation.

Equation of straight line in two points form:

Show that the equation of a st. line passes through two given point (x_1, y_1) and (x_2, y_2) is $y- y_1 = \frac{y_1 – y_2}{S_1 – S_2} (x – x_1)$



Let AB be a st. line passes through two given point $Q(x_1, y_1)$ and $R (x_2, y_2)$. Again let P be the any point on the same line whose co-ordinate is (x,y) and < BAX = q

Draw □r on x – axis from Q to QL R to RM P to PN and R to RT on QL and Q to QS on BN Alos < Q RT = q = < BQS

$RT = ML = OL- OM = x_1 – x_2$

QT = QL – LT = QL - RM = $y_1 - y_2$
QS = LN = ON – OL = $x - x_1$
PS = PN – SN = PN – QL = $y - y_1$
Now from rt. angled □ QRT , tanq = $\frac{QT}{RT} = \frac{y_1 - y_2}{S_1 - S_2}$ ------- (1)

Similarly from rt. angled □ PQS, tanq = $\frac{PS}{QS} = \frac{y - y_1}{S - S_1}$ ----- (2)

Now comparing the eq^n (1) and (2) we have

$$\frac{y - y_1}{S - S_1} = \frac{y_1 - y_2}{S_1 - S_2}$$

or, $y - y_1 = \frac{y_1 - y_2}{S_1 - S_2}(x - x_1)$. This is the required equation.

Equation of straight line in intercept form:

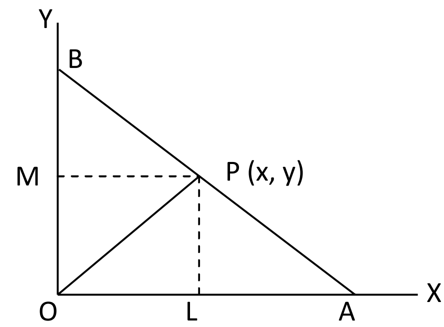
Show that equation of the st. line making intercepts a and b on the co-ordinate axes is x/a +y/b = 1

Let AB be a st. line which cuts x-axis at a point A and

y- axis at a point B respectively and let intercepts

OA = a and OB = b

Let P be the any point on the line AB



Whose co-ordinate is (x,y). Draw □ r from P to PL on x-axis and P to PM on y-axis. Join OP

\PL = y, and PM = x

Now □ AOB = □AOP + □BOP

□ ½ OA . OB = ½ OA. PL + 1/2 OB ´ PM

□ ½ a.b = ½ a.y + ½ bx

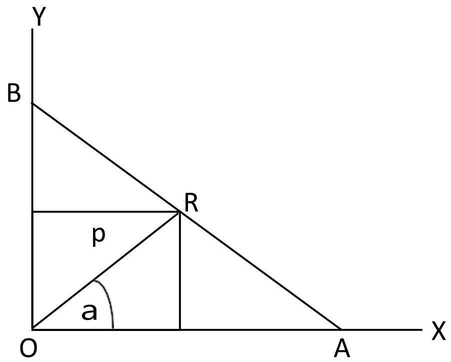
Divided both sides by ab

$\frac{S}{a} + \frac{y}{b} = 1.$

Since P be the any point on AB so the equation $\frac{S}{a} + \frac{y}{b} = 1$ is always true all points on AB.

Equation of straight line in normal form:

To obtain the equation of a st. line in the form of x cos a + y sin a = p. Where p is the perpendicular from the origin on the line and a is the angle which this perpendicular makes with x –axis.



Let AB be a st. line which cuts
the x and y –axis at A and B respectively
OR be a $\perp$r on the line
i.e. OR = p and < ROA = a

$\therefore < POB = \left(\frac{\pi}{2} - a\right)$

In Δ AOP ; Sec a = $\frac{OA}{OR} = \frac{OA}{p}$ or, OA = p sec a

Similarly in Δ OPB, Sec $\left(\frac{\pi}{2} - a\right) = \frac{OB}{OR} = \frac{OB}{P}$ or, OB = p cosec a

Since OA and OB are the intercepts of the line so the eq^n of the line in the form of intercepts will be

$$\frac{x}{OA} + \frac{y}{OB} = 1$$

Or, $\frac{x}{p\ seC\ a} + \frac{y}{p\ CoseC\ a} = 1$

Or, $\frac{x\ Cos\ a}{p} + \frac{y\ sin\ a}{p} = 1$

Or, x cos + y sin a = p This is the required equation.

General equation of the straight Line:

ax +by + c = 0 is the general equation of the straight line, where a, b, c are constant quantity.

Show that every equation of the first degree in x & y represents a straight line.

Proof:

Let the general equation of a st. Line of the 1st degree in x and y is ax + by + c = 0 ---------(1)

Let P (x_1 , y_1), Q (x_2, y_2) and R (x_3, y_3) be the three points on the locus of the line

ax + by + c =0

So Point P, Q, R satisfies the equation (1)

$ax_1 + by_1 + c = 0$ ---------(2)

$ax_2 + by_2 + c = 0$ -------- (3)

$ax_3 + by_3 + c = 0$ ----------(4)

Subtract eq^n (2) from (3)

$a(x_2 - x_1) + b(y_2 - y_1) = 0$

or, $\frac{S_2 - S_1}{y_2 - y_1} = -\frac{b}{a}$

or, $\frac{y_2 - y_1}{S_2 - S_1} = -\frac{a}{b}$ --------- (5)

Again subtract equation 3 from 4

$a(x_3 - x_2) + b(y_3 - y_2) = 0$

or, $\frac{y_3 - y_2}{S_3 - S_2} = -\frac{a}{b}$ -----(6)

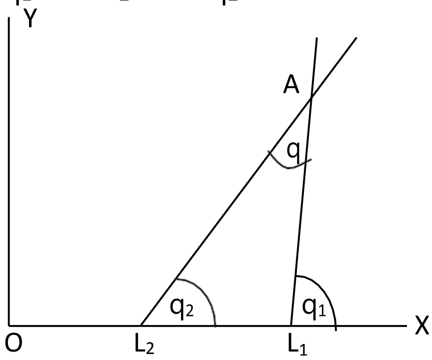
Now from equation (5) and (6) we have

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$ ----------- (7)

Hence this proves that □ PQR = 0
It shows that point P, Q, R are collinear
Finally equation (1) represents a st. Line.

Angle between two straight lines:

Let AL_1 and AL_2 be two st. Lines whose equation $y = m_1x + c_1$ and
$Y = m_2 x + c_2$
Which makes angle q_1 and q_2 with axis respectively where $m_1 = \tan q_1$ and $m_2 = \tan q_2$
Let q be the angle between these two lines i.e. $< L_1 AL_2 = q$



From figures $< L_2AL_1 = AL_1X - AL_2 X$
Or $q = q_1 - q_2$
If q be the supplement angle $L_1 AL_2$ then
$q = q_2 - q_1$ $\qquad q + q_2 + 180 + q_1 = 360$
If we combine these two
$q = \pm (q_1 - q_2)$
Taking tan both sides

$\tan q = \pm \tan(q_1 - q_2) = \frac{\tan q_1 - \tan q_2}{1 + \tan q_1 \tan q_2} = \frac{m_1 - m_2}{1 + m_1 m_2}$ ---- (A)

If equation of the st. Line are in the form of
$a_1 x + b_1 y + c_1 = 0$ ---------- (1)
$a_2 x + b_2 y + c_2 = 0$ ---------(2)
or, $\frac{a_1}{b_1} x + y + \frac{c_1}{b_1} = 0$ or, $y = -\frac{a_1}{b_1} - \frac{c_1}{b_1}$ -------(3)

and $y = -\frac{a_2}{b_2} x - \frac{c_2}{b_2}$ -----------(4)

Eq^n (3) & (4) becomes in the form of $y = mx$

Putting the value of $m_1 = -\frac{a_1}{b_1}$ and $m_2 - \frac{a_2}{b_2}$ in eq^n (A) we get,

$$\text{Tan}q = \frac{-\frac{a_2}{b_2} + \frac{a_1}{b_1}}{1 + \frac{a_1}{b_1}\frac{a_2}{b_2}} = \frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2}$$

$$\tan q = \frac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2} \quad \text{---- (B)}$$

1. If the two lines are parallel (When eq^n of the line in to term of $y = mx + c$

   $q = 0$

   $\tan 0 = \frac{m_1 - m_2}{1 + m_1 - m_2}$ or, $0 = \frac{m_1 - m_2}{1 + m_1 - m_2}$ $\backslash\ m_1 = m_2$

2. If two lines are 1r to each other means $q = \frac{h}{2}$

   Or, $\tan \frac{h}{2} = \frac{m_1 - m_2}{1 + m_1 - m_2}$

   Or, $\alpha = \frac{m_1 - m_2}{1 + m_1 - m_2}$

   Or, $\frac{1}{0} = \frac{m_1 - m_2}{1 + m_1 - m_2}$ or, $1 + m_1 - m_2 = 0$ or, $m_1 m_2 = -1$

   IF Eq^n s are in the form of

   $a_1 x + b_1 y + c_1 = 0$, $a_2 x + b_1 y + c_2 = 0$

   These two lines are parallel if $a_1 b_2 - a_1 b_2 = 0$ □ $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

   and these two lines are perpendicular when $a_1 a_2 = - b_1 b_2$

   $a_1 a_2 + b_1 b_2 = 0$

   - To find the eq_n of a st. line which is parallel to the given line- we remain as such the co-efficient of x and y but put any other constant term.
   - To find the eq^n of the line which is perpendicular to given lines. In this we interchange the coefficient of x and y and also change the sign between the x and y and put any other constant term.

Point of intersection of two straight lines:

Let $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ be the two st. lines and (α, β) be the co-ordinates of points of intersection.

$a_1 x + b_1 y + c_1 = 0$ ---------(1)

$a_2 x + b_2 y + c_2 = 0$ -------(2)

After solving the eq^n (1) & (2) we get

$$\frac{}{b_1 c_2 - b_2 c_1} = \frac{\beta}{a_2 c_1 / c_2 a_1} = \frac{1}{a_1 b_2 / a_2 b_1}$$

$$\backslash\ \alpha = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 + a_2 b_1} \ \&\ \beta = \frac{a_2 c_1 - a_1 c_2}{a_1 b_2 + a_2 b_1}$$

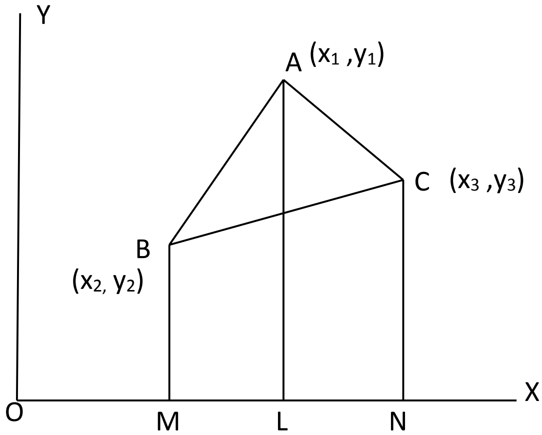
If $a_1 b_2 - a_2 b_1 = 0$ then these two lines are parallel.

Area of Triangle

To find the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3)

Let ABC be a triangle and the co-ordinates of vertices A, B, C are (x_1, y_2) and (x_3, y_3) respectively.

Draw ⊥r on x-axis from A to AL, B to BM and C to CN



∴ Δ ABC = Trapezium ABML + Trapezium ALNC – Trapezium BMNC

Since Area of trapezium = ½ [Addition of Parallel sides] × [Perpendicular distance between them]

∴ Δ = ½ (BM +AL)XML + ½ (AL +CN) LN – ½ (BM + CN) MN

$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

$= \frac{1}{2}(y_2 x_1 + y_2 x_2 + y_1 x_1 - y_1 x_2 + y_1 x_3 - y_1 x_1 + x_3 y_3 - y_3 x_1 - y_2 x_3 + x_2 y_2 - x_3 y_3 + x_2 y_3)$

$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Condition that three points are collinear

If area of the Δ will be zero then three points are collinear.

Area of Quadrilateral:

To find the area of a quadrilateral divide the quadrilateral in two triangle and add the area of the two triangles. The method of dividing into two triangles to find out its area is known as triangulation.

Let ABCD is a quadrilateral

The Area of Quad ABCD = (Area of Δ ABD) + (Area of Δ BCD)

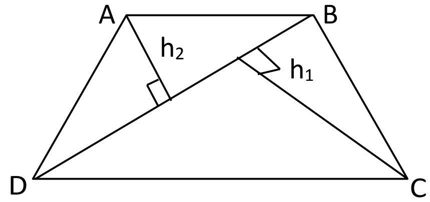
Area of quadrilateral ABCD = ½ d $(h_1 + h_2)$ where d is the diagonal and h1 and h_2 are the heights of quadrilateral.

Area of quadrilateral ABCD

$= (\frac{1}{2} \times DB \times h_1) + (\frac{1}{2} \times DB \times h_2)$

$= \frac{1}{2} \times DB \times (h_1 + h_2)$

$= \frac{1}{2} \times d \times (h_1 + h_2)$



Where d denotes the length of the diagonal DB

If h_1 = 7cm

h_2 = 5 cm

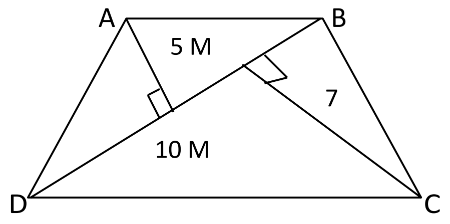
BD = 10 cm

Then area of ABCD

= ½ ´ 10 ´ (5+7)

= ½ ´ 10 ´ 12

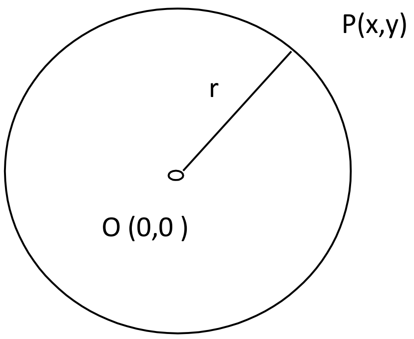
= 60



Circle

Circle is the locus of a moving point in the plane which moves in such a way that its distance from a fixed point is always constant (The fixed point is called centre of circle and constant distant is called radius of the circle)

Equation of circle whose centre is (0, 0) and radius is "r"



Since,

$OP=\sqrt{(x-0)^2+(y-0)^2}$

$\Rightarrow \sqrt{(x-0)^2+(y-0)^2} = r$

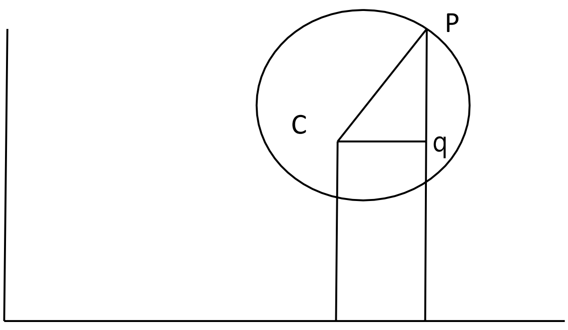
$\Rightarrow (x-0)^2 + (y-0)^2 = r^2$

$\Rightarrow \quad x^2 + y^2 = r^2$

This is the require equation of the circle whose centre is (0,0) and radius r

Equation of the circle whose is (0,0) and radius is a

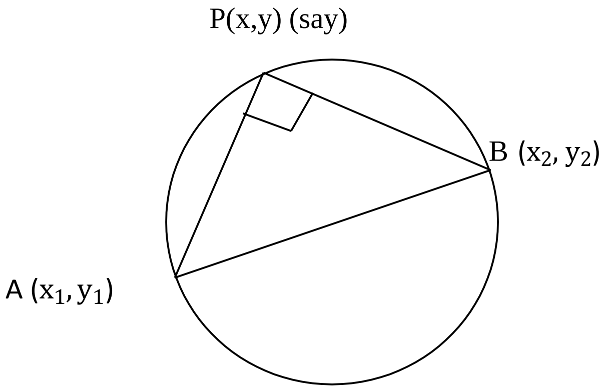
i.e. $x^2 + y^2 = a^2$



$(x-\alpha)^2 + (y-\beta)^2 = r^2$

The general eq^n circle is $x^2 + y^2 + 2gx + 2fy+c = 0$ whose centre is (-g , -f) and radius is $\sqrt{g^2+f^2-c}$

Equation of circle whose diameters is the line joining two points (x_1, y_1) and (x_2, y_2) :



From geometry

We have AP□ PB

Therefore,

Slope of AP. Slope of PB = -1

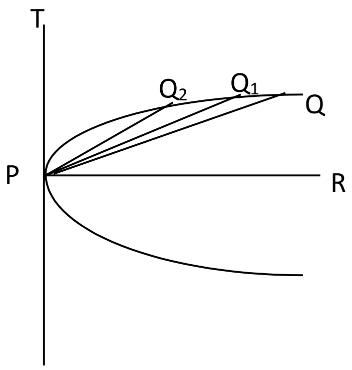
Or, $\frac{(S_1-S)}{(y_1-y)} \times \frac{(S_2-S)}{(y_2-y)} = -1$

Or, $(x_1-x)(x_2-x) = -(y_1-y)(y_2-y)$

Or, $(x_1-x)(x_2-x) + (y_1-y)(y_2-y) = 0$ This is the equation of required circle

Point 4,5,6

Tangent, Normal & Point Contact



PQ = chord

PT = Tangent

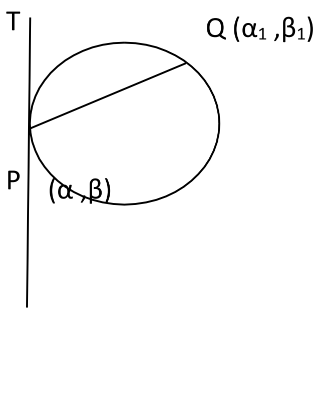
PR = Normal

Point P is known as point contact.

To find eq^n of the tangent to the circle $x^2 + y^2 = a^2$ at the point (α, β) on its circumference.

Let P be the any point on the circle whose co-ordinate is (α, β) and Q be the another point on the curve which is very nearest to P and whose co-ordinate is (α_1, β_1)

The eq^n of the chord PQ



$y - \beta = \frac{\beta - \beta_1}{\alpha - \alpha_1}(x - \alpha)$ -----------(1)

Since (α, β) & (α_1, β_1) are situated on the circle $x^2 + y^2 = a^2$

Hence $\alpha^2 + \beta^2 = a^2$

$\alpha_1^2 + \beta_1^2 = a^2$

Now $(\alpha^2 - \alpha_1^2) + (\beta^2 - \beta_1^2) = 0$

or, $(\alpha - \alpha_1)(\alpha + \alpha_1) + (\beta - \beta_1)(\beta + \beta_1) = 0$

or, $\frac{\beta - \beta_1}{\alpha - \alpha_1} = \frac{\alpha + \alpha_1}{\beta + \beta_1}$

we put this value in eq^n we get

$y - \beta = - \frac{\alpha + \alpha_1}{\beta + \beta_1} (x - \alpha)$ ---------(2)

Now As Q tends to P, $\alpha_1 \to \alpha$ & $\beta_1 \to \beta$ chord PQ tends to the tangent on Point we put $\alpha_1 = \alpha$, $\beta_1 = \beta$ eq^n (2) we get the eq^n of tangent PT

□ $y - \beta = - \frac{\alpha}{\beta} (x - \alpha)$

□ $(y - \beta) = -\alpha (x - \alpha)$

□ $\beta y - \beta^2 = -\alpha x + \alpha^2$

□ $\beta y + \alpha x = \alpha^2 + \beta^2$

□ $x\alpha + y\beta = a^2$ [Since ($\alpha_1\beta$) is situated on the circle]. This is required eq^n

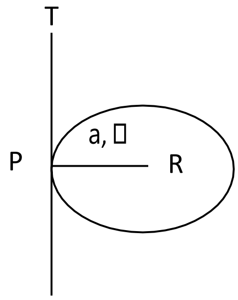
To find the equation of the normal at the point (α, β) of the circle $x^2 + y^2 = a^2$

Let P be the point on the circle whose co-ordinate is (α, β) PR be line which passes through (α, β) and it is also □r on the tangent PT. We have to find eq^n of PR.

We know that the eq^n of tangent on the circle $x^2 + y^2 = a^2$

$x\alpha + y\beta = a^2$ Divided both sides by β

$y = - \frac{}{\beta} x + \frac{\alpha^2}{\beta}$ -----------(1)



The eq^n of the st. line which passes through a point P (α, β) is $y - \beta = m (x - \alpha)$ ----- (2)

If eq^n (2) is normal then it means □r to eq^n (1) In this situation

$m_1 m_2 = -1$

i.e. $- \frac{}{\beta} \times m = -1$

i.e. $m = \frac{\beta}{}$

Putting the value of m in eq^n (2) we get

$y - \beta = \frac{\beta}{} (x - \alpha)$

□ $y \alpha - \alpha\beta = \beta x - \beta x$

or, $\beta x - \alpha y = 0$ → This is the required eq^n.

FUNCTION:

Definition of a function: Let A and B be two non-empty sets, then a rule ‘f’ which associates each element of A with a unique element of B is called a mapping or function from A to B. If A is a mapping from A to B, we write f : A □ B (read as f is a mapping from A to B).

Note: - If *f* associates x Î A to y Î B, then we say that y is the image of the element x under the map f and denote it by f(x) and we write y = f (x). The element x is called the pre-image or in verse- image of y.

Domain and Range of a function: The set A is called the domain of the map and the set B is called the co-domain. The set of the images of all the element of A under the map f is called the range of and is denoted by f (A).

Thus range of f i.e.f(A) = {f(x) : x**Î** A}

learly, f (A) □ B. If f is into & f(A) = B [when f is onto]

Domain or Range: If variable x takes all numerical values between ‘a’ and ‘b’ then totality of all values of x is known as domain or range of x i.e. a □ x □ b

Function: Let x and y be the set of two variables. If definite value of y can be obtained for the every value of x in the domain then y is known as function of variable x and we write

y = f(x) example in $y = 1+2x^2$]

If we put x = 0 , 1, 2, 3 4 then y = 1, 3, 5, 19, 33 respectively.

F: A → B (read as f is a mapping from A to B)

1. **Odd function:** A function f(x) is odd if f(-x) = -f(x), i.e. f(-x) + f(x) = 0
2. **Even function:** A function f(x) is even if f(-x) = f(x), i.f. f(-x) – f(x) = 0

Properties of odd and even function

1. a constant function us an even function
2. a zero function is both an odd and an even function.
3. D.C.of an odd function is an even function and d.c. of even function is an odd function.

Polynomial function: A function f : R→ R defined by $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_n x^n$ [When $a_0, a_1, \dots, a_n$ **Î** R]

Jacobian of Transformation: Transformation two dimensions random variables.

$$J = \frac{\P (S,y)}{\P (u,v)} = \begin{vmatrix} \frac{\P S}{\P u} & \frac{\P y}{\P u} \\ \frac{\P S}{\P v} & \frac{\P y}{\P v} \end{vmatrix}$$

Let the rv's X and Y with joint pat fxy (x,y) be transformed to the r.v.s u and v by the transformation u= u (x,u), v= (x,y) where u and v ate continuously differentiable function for which

$$J = \frac{\partial (S,y)}{\partial (u,v)} \cdot \begin{vmatrix} \frac{\partial S}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial S}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

is either > 0 or< 0 throughout the (x,y) plane so that the inverse transformation is uniquely given by x =x (u,v), y = y(u,v)

Constant Function: Let c be a fixed real number then the function f: R→R (Function f from R to R) is said to be constant function if f(x) = c, for every x ∈ R

Identity function: A map f : R →R is said to be an identity function iff f (x) = x,∀ x∈R

Note: [Identity function is sometimes also called the function x

Domain of identity function = R

Range of identity function = R]

Even function:A map of f : A → B is said to be an even function iff f(-x) = f(x) for all x∈ A.

Ex. 1. f (x) = x^2 for all x ∈ R 2. f (x) = cosx for all x ∈ R

Odd function: A map f : A → B is said to be odd function iff f (-x) == -f(x) for all x ∈ A.

Ex. 1. f(x) x^3, ∀x ∈ R 2. f(x) = sinx, ∀x ∈ R.

LIMIT

Let us consider the sequence

$U_n = \frac{1}{n}$ If we put n = 1, 2,3..... 10000

$U_1 = 1$

$U_2 = \frac{1}{2}$

$U_3 = \frac{1}{3}$

⋮ ⋮

$U_{1000} = \frac{1}{1000}$

Here we see that as n → ∞; $\frac{1}{n} \to 0$, this zero is known as limit of $\frac{1}{n}$ or limiting value of $\frac{1}{n}$ is o

→ read as 'tends to'

Definition of left hand limit: We say that left hand limit of f(x) as x tends to a exists and is equal to l_1 if as x approaches a, always remaining less than a, the values of f(x) approach a definite unique real number l_1. In other words if for every ∈ > 0, however small exists δ > 0 such that

$l_1 - \epsilon < f(x) < l_1 + \epsilon$ i.e. $|f(x) - l_1| < \epsilon$ for all x for which a - δ < δ < x <a.

In this case we write $\underset{S \to a-0}{Lt} f(x) = l_1$

Definition of right hand limit: We say that right hand limit of f(x) as x tends to a exists and is equal to l_2 if as x approaches a always remaining greater than a, the values of f(x) approach a definite unique real number l_2. In other words if for every Î > 0, however small, there exists d > 0, such that

l_2 - Î <f (x) < l_2 + Î i.e. | f(x) - l_2 | < Î for all x for which a < x < a + d

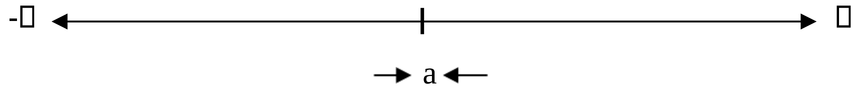
In this case we write $\underset{S\to a+0}{\text{Lt}}$ f(x) = l_2

Definition of limit: We say that limit of f(x) as x tends to a exists and is equal to a real number l if as x approaches a (through the values less than or greater than a) the values of f(x) approach a definite unique real number l. In other words if for every Î > 0, however small, there exists d > 0, such that

l - Î <f(x) < l + Î i.e.|f(x) – l | < Î for all x for which a-d < x< a + d i.e.| x - a| < d

In this case we write $\underset{S\to a}{\text{Lt}}$ f(x) = l

Thus the statement $\underset{S\to a}{\text{Lt}}$ f(x) = 1 means that the values of f(x) will approach the number l or are equal to l as the values of x approach the number a from either direction.



Important properties of limit: If $\underset{S\to\alpha}{\text{Lt}}$ f(x) and $\underset{S\to\alpha}{\text{Lt}}$ g(x) exist , then

1. $\underset{S\to a}{\text{Lt}}\{f(x) + g(x)\} = \underset{S\to a}{\text{Lt}}\{f(x) + \underset{S\to a}{\text{Lt}}\, g(x)$
2. $\underset{S\to a}{\text{Lt}}\{f(x) - g(x)\} = \underset{S\to a}{\text{Lt}}\, f(x) - \underset{S\to a}{\text{Lt}}\, g(x)$
3. $\underset{S\to a}{\text{Lt}}\{c.f(x)\} = c.\underset{S\to a}{\text{Lt}}\, f(x)$, where c is a constant
4. $\underset{S\to a}{\text{Lt}}\{f(x).g(x)\} = \{\underset{S\to a}{\text{Lt}}\, f(x)\}.\{\underset{S\to a}{\text{Lt}}\, g(x)\}$
5. $\underset{S\to a}{\text{Lt}}\left\{\frac{f(S)}{g(S)}\right\} = \frac{\underset{x\to a}{\text{Lt}}\, f(S)}{\underset{x\to a}{\text{Lt}}\, g(S)}$. Provided $\underset{S\to a}{\text{Lt}}\, g(x)$ □ 0. g(x) □ 0

Values and limits of a function, continuity: The values and limits of any function are not always equal to each other. The values and limits of a given function at any point are two separate things. These may be equal or may not be equal . If on any point the limits and values of a function are equal then that function known as **continuous function** and if theyare not equal then it is known as **discontinuous function**.

If any function f(x) is given which is defined for x = 0 then f(x) is known as continuous at x = a if

$$\underset{h\to 0}{Lt}\ f(a+h) = f(a) = \underset{h\to 0}{Lt}\ (a-h)$$

Example on continuity: Find the continuity of cosq **at** q **=0 ,** Here for q = 0 f(0) = cos 0 = 1

$$\lim_{h\to 0} f(o+h) = \lim_{h\to 0} \cos(o+h) = \cos 0 = 1$$

and $$\lim_{h\to 0} f(o-h) = \lim_{h\to 0} \cos(o-h) = \cos 0 = 1$$

Here all the conditions of continuity have been satisfied.So, cosq is continuous at q = 0

Formula: 1. $\text{Lt}_{S\to a} \frac{S^n - a^n}{S-a} = n\,a^{n-1}$

2. $\text{Lt}_{q\to 0} \frac{\text{Sin}\,q}{q} = 1$

3. $\text{Lt}_{q\to 0} \text{Cos}\, q = 1$

4. $\text{Lt}_{q\to 0} \frac{\tan q}{q} = 1$

Theorem: Prove that $\text{Lt}_{q\to 0} \frac{S^n - a^n}{S-a} = na^{n-1}$ for all rational values of n provided a is positive.

Proof: If we put x=a then function becomes in the $\frac{0}{0}$ form i.e. indeterminate form.

So, at x = a, solution is not possible. Hence we put x = a+h where h is constant quantity.

As x ⟶ a, h ⟶ 0

Now, L.H.S = $\text{Lt}_{h\to 0} \frac{(a+h)^n - a^n}{a+h-a} = \text{Lt}_{h\to 0} \frac{a^n(1+\frac{h}{a})^n - a^n}{h} = \text{Lt}_{h\to 0} \frac{a^n[1+\frac{h}{a}]^n - 1}{h}$

By Binomial theorem we have, $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \ldots\infty$

$$= \text{Lt}_{h\to 0} \frac{a^n[1+n\frac{h}{a}+\frac{n(n-1)}{\lfloor 2}(\frac{h}{a})^2 + \frac{n(n-1)(n-2)}{\lfloor 3}+(\frac{h}{a})^3 \ldots\ \text{higher power of h} - 1]}{h}$$

$$= \text{Lt}_{h\to 0} a^n.k.\frac{h}{a} \times \frac{1}{h} + \text{Lt}_{h\to 0} \frac{n(n-1)}{\lfloor 2} . \frac{h}{a^2} + \frac{n(n-1)(n-2)}{\lfloor 3} + \frac{h^2}{a^3} + \cdots \text{higher power of } h$$

$= a^n . \frac{n}{a} + 0 + 0 + 0 + \ldots = na^{n-1}$ Proved. \ L.H.S. = R.H.S.

Examples: 1. Find the value of $\text{Lt} \frac{\text{Sin}\,q}{} = (\text{Lt} \frac{\text{Sin}\,q}{}) \times a\,q / (\text{Lt} \frac{\tan q}{}) \times bq = \frac{1 \times a\,q}{} =$

$q \to 0$ t an$q$ $q \to 0$ a q $q \to 0$ b q $1 \times b\, q$

$\frac{q}{b}$ Ans.

2. Find $\underset{S \to 0}{\text{Lt}} \frac{\text{Tan } S\ 0}{S}$ We know that $180^0 = \pi$ rad. \ $x^0 = x \frac{h}{180}$ rad.

$$= \underset{S \to 0}{\text{Lt}} \frac{\tan \frac{\pi x}{180}}{S} = \underset{S \to 0}{\text{Lt}} \frac{\tan \frac{\pi x}{180}}{\frac{\pi x}{180} \times \frac{180}{\pi}} = \frac{h}{180} \text{ Ans.}$$

Prove that

i. limit of $\frac{S(\cos S+\cos 2S)}{\sin S}$ as $x \to 0$ = ,[$\underset{S\to 0}{Lt}\ \cos x$ + $\underset{S\to 0}{Lt}\ \cos 2x$] / $\underset{S\to 0}{Lt}\ \frac{\sin S}{S} = \frac{1+1}{1}$ = 2 Ans.

ii. $\underset{S\to 0}{Lt}\ \frac{\cos S-\cos 3S}{S(\sin 3S-\sin S)} = 2$

iii. $\underset{S\to 0}{Lt}\ \frac{\tan S+\sin S}{1-\cos S} = 0$, Proof. $\underset{S\to 0}{Lt}\ \frac{\frac{\sin x}{\cos x}-\sin S}{1-\cos S} = \underset{S\to 0}{Lt}\ \frac{\sin S\,(1-\cos S)}{\cos S\,(1-\cos S)} = \underset{S\to 0}{Lt}\ \tan x = \underset{S\to 0}{Lt} = \frac{\tan S}{S} \times = 0$

iv. $\underset{S\to 0}{Lt}\ \frac{\tan S-\sin S}{S^3} = \frac{1}{2}$

v. $\underset{S\to y}{Lt}\ \frac{\tan S-\tan y}{S-y} = \text{Sec}^2 y$

Solution: If we put x = y then given function becomes in the form of $\frac{0}{0}$ so we put x = y +h

As x □ y $h\to 0$

$$= \underset{h\to 0}{Lt}\ \frac{\tan(Y+h)-\tan y}{y+h-y} = \underset{h\to 0}{Lt}\ \frac{\frac{\sin(y+h)}{\cos(y+h)}-\frac{\sin y}{\cos y}}{h} = \underset{h\to 0}{Lt}\ \frac{\text{Sin}(y+h)\cos y-\text{Cos}(y+h)\sin y}{\cos y\ \text{Cos}(y+h)h}$$

$$= \underset{h\to 0}{Lt}\ \frac{\text{Sin}(y+h-y)\cos y}{\cos y\ \text{Cos}(y+h)h} = \underset{h\to 0}{Lt}\ \frac{\sinh}{h} \times \frac{1}{\underset{h\to 0}{Lt}\ \text{Cos}(y+h)} \times \frac{1}{\underset{h\to 0}{Lt}\ \text{Cos}\,y} = 1 \times \frac{1}{\cos y} \times \frac{1}{\cos y} = \text{Sec}^2 y$$

vi. $\underset{S\to □}{Lt}\ \frac{S\,\text{Sin}\alpha-\alpha\text{Sin}S}{S-\alpha} = \text{Sin}\,\alpha-\alpha\cos\alpha$ Solution: we put x = α +h

$$= \underset{h\to 0}{Lt}\ \frac{(\alpha+h)\,\text{Sin}\alpha-\alpha\,\text{Sin}(\alpha+h)}{\alpha+h-\alpha} = \underset{h\to 0}{Lt}\ \frac{\alpha\,\text{Sin}\alpha+h\sin\alpha-\alpha\,\text{Sin}\alpha\,.\text{Cos}\,h-\alpha\,\text{Cos}\,\alpha\sin h}{h}$$, After simplication

$$= \alpha\sin\alpha\ \underset{h\to 0}{Lt}\ \frac{2\,\text{Sin}^2\frac{h}{2}}{(\frac{h}{2})^2} \times \frac{h^2}{4} + \text{Sin}\,\alpha-\alpha\cos\alpha\ \underset{h\to 0}{Lt}\ \frac{\text{Sinh}}{h} = 0 + \text{Sin}\,\alpha-\alpha\cos\alpha \times 1$$

= Sin α–α cosα Hence,proved.

vii. $\underset{S\to 0}{Lt}\ \frac{1-\text{cosec}\,S}{S^2}$ $(K \neq 0) = \frac{K^2}{2}$

viii. $\underset{S\to 0}{Lt}\ \frac{\tan S-\text{Sin}S}{S^3} = 2$

ix. $\underset{S \to 0}{Lt} \dfrac{\cos 7S - \cos 9S}{\cos 3S - \cos 5S} = 2$

x. $\underset{S \to 0}{Lt} \dfrac{\operatorname{cosec} S - \operatorname{Cot} S}{S} = 2$

xi. $\underset{S \to 0}{Lt} \dfrac{S\ (\cos S + \cos 2S)}{\operatorname{Sin} S} =$ ____________ ____

xii. $\underset{S\to 0}{Lt} \frac{3\sin S + \sin 3S}{S(\cos 2S - \cos 4S)} = \underset{S\to 0}{Lt} \frac{3\sin S - 3\sin S + 4\sin^3}{S[2\sin 3S - \sin S]} = \underset{S\to 0}{Lt} \frac{4\sin^2 s}{2\sin 3S} = 2 \times \underset{S\to 0}{Lt} \left(\frac{\sin S}{S}\right)^2 \underset{S\to 0}{Lt} \frac{\sin 3S}{3S} \times \frac{1}{2}$

$= \frac{2}{3}$ Ans.

xiii. $\underset{S\to ?}{Lt} \left[\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{n}{n^2}\right] = 1/3$

Indeterminate Form:

$\frac{0}{0}, \frac{?}{?}, 0.?, \infty - \infty, 1^{\infty}, 0^{\infty}, \infty^{0}$

L' Hospital's rule

If f(x) and g(x) are differentiable function at x = a

1. If f (a) = 0 and g(a) = 0

   Then $\underset{S\to a}{Lt} \frac{f(S)}{g(S)} = \underset{S\to a}{Lt} \frac{f'(S)}{g'(S)} = \underset{S\to a}{Lt} \frac{f^{FF}(S)}{g^{FF}(S)} = \ldots\ldots \underset{S\to a}{Lt} \frac{f^n(S)}{g^n(S)}$

2. If $\underset{S\to a}{Lt}$ f(x) = ? and $\underset{S\to a}{Lt}$ g(x) = ∞ then $\underset{S\to a}{Lt} \frac{f(S)}{g(S)} = \underset{S\to a}{Lt} \frac{f^F(S)}{g^F(S)} = \underset{S\to a}{Lt} \frac{f^{FF}(S)}{g^{FF}(S)} = \ldots$

   $= \underset{S\to a}{Lt} \frac{f^n(S)}{g^n(S)}$

In other words, we differentiate the numerator and denominator separately till the formation of $\frac{0}{0}$ or $\frac{\infty}{\infty}$ has not been closed.

1. Example: $\underset{q\to 0}{Lt} \frac{\sin q}{q} = \underset{q\to 0}{Lt} \frac{\text{d.c,of} \sin q}{\text{d.c,of} q} = \underset{q\to 0}{Lt} \frac{\cos q}{1} = \cos q = 1$

   L' Hospital's rule is applicable only when the form is $\frac{0}{0}$, $\frac{\infty}{\infty}$

2. Find $\underset{S\to 1}{Lt} \frac{S^7 - 2S^5 + 1}{S^3 - 3S^2 + 2} \left[\frac{0}{0} - \text{Form}\right]$

   $= \underset{S\to 1}{Lt} \frac{7S^6 - 10S^4}{S^3 - 3S^2 + 2}$ [By L' Hospital's rule]

   $= \frac{7-10}{3-6} = \frac{3}{3} = 1$ Ans.

3. $\underset{S\to \alpha}{Lt} \frac{S\sin\alpha - \alpha \sin S}{S - \alpha}$ Here x is variable & α is constant given function in the form of $\frac{0}{0}$

[By L' Hospital's rule]

$= \underset{S \to \alpha}{Lt} \frac{1 . \text{Sin}\alpha - \alpha \cos S}{1-0} = \text{Sik}\alpha - \alpha \cos x$ Ans.

4. $\underset{S \to \frac{\pi}{2}}{Lt} (\frac{h}{2} - x) \tan x$ This is $0, \infty$ Form

$= \underset{S \to \frac{\pi}{2}}{Lt} \frac{(\frac{\pi}{2} - x)}{\text{Cot } S} \; [\frac{0}{0}]$

$$= \underset{S \to \frac{\pi}{2}}{Lt} \frac{0-1}{-\operatorname{cosec}^2 S} = \underset{S \to \frac{\pi}{2}}{Lt} \sin^2 x = \left(\frac{\sin h}{2}\right)^2 = 1$$

5. $\underset{q \to \frac{\pi}{2}}{Lt} (\operatorname{Sec} q - \tan q) = \underset{q \to \frac{\pi}{2}}{Lt} \left(\frac{1}{\cos q} - \frac{\sin q}{\cos q}\right)$

$$= \underset{q \to \frac{\pi}{2}}{Lt} \frac{0-1}{-\operatorname{cosec}^2 S} = \underset{q \to \frac{\pi}{2}}{Lt} \frac{0-(+\cos q)}{-\sin q} = \underset{q \to \frac{\pi}{2}}{Lt} \cot q = 0 \text{ Ans.}$$

6. Find $\underset{S \to 0}{Lt} \frac{\sqrt{1+S}-\sqrt{1-S}}{S}$ $[\frac{0}{0}$ Form]

$$= \underset{S \to 0}{Lt} \frac{(1+S)^{\frac{1}{2}} - (1-S)^{\frac{1}{2}}}{S} = \underset{S \to 0}{Lt} \frac{\frac{1}{2}\frac{1}{(1+x)^{\frac{-1}{2}}} - \frac{1}{(1+x)^{\frac{-1}{2}}}(-1)}{S} = 1 \text{ Ans.}$$ [By L Hospital's rule]

7. Find $\underset{S \to 0}{Lt} \frac{e^x - 1 - S}{S^2} = \underset{S \to 0}{Lt} \frac{1+S+\frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots - 1 - S}{S^2} = \underset{S \to 0}{Lt} \frac{\frac{S}{1}\left[\frac{1}{\lfloor 2} + \frac{x}{\lfloor 3} + \frac{x^2}{\lfloor 3} + \cdots\right]}{S^2} = \frac{2}{\lfloor 2} = \frac{1}{2}$ Ans.

8. Show that $\underset{S \to 0}{Lt} \frac{(1+S)^n - 1}{S} = n$

9. $\underset{S \to 0}{Lt} \frac{a - \sqrt{a^2 - S^2}}{S^2} = \frac{1}{2a}$

Differentiation

If the value of any variable x changes from x_1 to x_2 then quantity x_2-x_1 is known as increment of x. This small increment is represented by dx (Delta x) d x is not product of d and x but it is only infinitesimal increment in x

Let y = f (x) --- (1)

Y + dy = f(x+dx) --- (2)

We subtract (1) from (2)

dy = f(x+dx)– f(x)

or, $\underset{dx \to 0}{Lt} \frac{dy}{dS} = \underset{dx \to 0}{Lt} \frac{f(x+dx)}{dS} - \underset{dx \to 0}{Lt} \frac{f(x)}{dS}$

or, $\frac{dy}{dS} = \underset{dx \to 0}{Lt} \frac{f(x+dx) - f(S)}{dS}$

$\frac{dy}{dS}$ is known as differential coefficient of y w.r.t.x

$\frac{dy}{dS}$ is a fractional it can be separated while $\frac{dy}{dS}$ is not fractional it cannot be separated but it is the limiting value of $\frac{dy}{dS}$

Differential coefficient of any function is also known as derivative.

1. Find the d.c. of x^n w.r.t. x from the lst Principle

   Let $Y = x^n$ --------(1)

   $Y + dy = (x + dx)^n$ ----------- (2)

Now,(2) –(1)

$dy = (x + dx)^n - x^n = x^n(1 + \frac{dS}{S})^n - x^n$

$= x^n [(1 + \frac{dS}{S})^n - 1]$

or, $\frac{dy}{dS} = \frac{S^n[1 + n\frac{dx}{x} + \frac{n(n-1)}{L2}(\frac{dx}{x})^2 + \dots \text{higher power of } dS - 1]}{dS}$

or, $\underset{dx \to 0}{Lt} \frac{dy}{dS} = \underset{dx \to 0}{Lt}\ x^n . n. \frac{dS}{S} \times \frac{1}{dS} + \underset{dx \to 0}{Lt} \frac{n(n-1)}{L2} \frac{dS}{S^2} + \dots\dots$

or, $\frac{dy}{dS} = k\ x^{n-1}$

2. Find the d.c. of Sinx w.r.t. x from the 1st Principle

Let y = sin x------------(1)

y + dy = sin (x + dx)------------(2)

or, dy = sin (x + dx) – sinx = $2 \cos\frac{S + dS + S}{2} . \sin\frac{dS}{2}$

or, $\frac{dy}{dS} = 2 \cos (x + \frac{dx}{2}) . \frac{\sin\frac{dx}{2}}{\frac{dx}{2} \times 2}$

or, $\underset{dx \to 0}{Lt} \frac{dy}{dS} = 2 \underset{dx \to 0}{Lt} \cos (x + \frac{dx}{2}) \times \underset{dx \to 0}{Lt} (\frac{\sin\frac{dx}{2}}{\frac{dx}{2}}) \times \frac{1}{2}$

or, $\frac{dy}{dS} = \cos x.$

3. Di fferent i at e t anx w. r. t . x from t he 1st Pri nci ple

Let y =t an x-------(1)

y + dy =t an (x + dx)------(2)

or, dy= t an (x +dx) – t an x = $\frac{\sin (x + dx)}{\cos(x + dx)} - \frac{\sin x}{\cos x} = \frac{\sin (x + dx)\cos x - \cos(x + dx) \sin x}{\cos x \cos(x + dx)}$

$= \frac{\sin (x + dx - x)}{\cos S \cos (x + dx)} = \text{Sik } dx \times \frac{1}{\cos S \cos (x + dx)}$

Di vi de bot h s i des dbxyand t aki ng li mi t daxs→0

$\underset{x \to 0}{Lt} \frac{dy}{dS} = \underset{dx \to 0}{Lt} \frac{\sin dx}{dS} \times \frac{1}{\cos x} = \underset{dx \to 0}{Lt} \frac{1}{\cos(x + dx)}$

or, $\frac{dy}{dS} = 1 \times \frac{1}{\cos x} \times \frac{1}{\cos x} = \sec^2 x$ Ans .

4. Di fferent i at e Log x w. r. t . from t he li s t Pri nci ple.

Let y =log x---------------(1)

Y + dy =log (x + dx)

------------------------------(2)

From eq^{n} (1) and (2) we have

$$dy = \log(x + dx) - \log x = \log\left(\frac{x + dx}{x}\right) = \log\left(1 + \frac{dx}{x}\right) = \frac{dS}{S} - \frac{\left(\frac{dx}{x}\right)^2}{2} + \frac{\left(\frac{dx}{x}\right)^3}{3} - + \ldots\ldots\ldots \text{ higher}$$

power of dx

We divide both sides by dx and taking limit both sides as $dx \to 0$

$$\underset{dS\to 0}{Lt}\frac{dy}{dS} = \underset{dS\to 0}{Lt}\frac{dx}{x} \times \frac{1}{dx} - \underset{dS\to 0}{Lt}\frac{(\frac{dx}{x})^2}{dx.2} + \text{............ higher power of } dx$$

$$\therefore \frac{dy}{dS} = \frac{1}{x} + 0 + 0 + \ldots = \frac{1}{x}$$

5. Differentiate e^x w. r. t. x from the first Principle

Let $y = e^x$ ----------(1)

$y + dy = e^{x+dx}$------------ (2)

From (1) and (2) $dy = e^{x+dx} . e^x = e^x [e^{dx} - 1]$

$= e^x [1 + dx + (\frac{dS}{\lfloor 2})^2 + (\frac{dS}{\lfloor 3})^3 + \text{higher power of } dx - 1]$

$$\underset{dx\to 0}{Lt}\frac{dy}{dS} = \underset{dx\to 0}{Lt} e^x + e^x \underset{dx\to 0}{Lt}\frac{dS}{\lfloor 3} + e^x \underset{dx\to 0}{Lt}(\frac{dS}{\lfloor 3})^2 + \ldots\ldots\ldots\ldots$$

$$\therefore \frac{dy}{dS} = e^x$$

Summary: 1. If $y = x^n$ then $\frac{dy}{dS} = nx^{n-1}$

2. If y = sin x then $\frac{dy}{dS}$ = cos x

3. If y = cos x then $\frac{dy}{dS}$ = - sin x

4. If y = tan x then $\frac{dy}{dS} = \sec^2 x$

5. If y = cot x then $\frac{dy}{dS} = -\operatorname{cosec}^2 x$

6. If y = cosec x then $\frac{dy}{dS}$ = - cot x. cosec

7. If y = secx then $\frac{dy}{dS}$ = sec x.tan x

8. If y = log x then $\frac{dy}{dS} = \frac{1}{S}$

9. If $y = e^x$ then $\frac{dy}{dS} = e^x$

6. Differentiate $\sqrt{\text{Sik x}}$ form the list Principle

Let $y = \sqrt{\text{Sik x}}$ -----------(1)

$Y + dy = \sqrt{\text{Sik }(x + dx)}$ --------- (2)

$$dy = (\sqrt{\text{Sik }(x + dx)} - \sqrt{\text{Sik x}}) \times \frac{\sqrt{\text{Sin }(S + dS)} + \sqrt{\text{Sin S}}}{\sqrt{\text{Sin }(S + dS)} + \sqrt{\text{Sin S}}}$$

$$= \frac{\sqrt{\text{Sin}(S+dS)} - \sqrt{\text{Sin } S}}{\sqrt{\text{Sin}(S+dS)} + \sqrt{\text{Sin } S}} = \frac{2\,\text{Cos}\left(\frac{x+dx}{2}\right)\text{Sin}\,\frac{dx}{2}}{\sqrt{\text{Sin}(S+dS)} + \sqrt{\text{Sin } S}}$$

$$\underset{x\to 0}{\text{Lt}}\frac{dy}{dx} = \frac{2 \times \underset{x\to 0}{\text{Lt}}\cos\left(\frac{x+dx}{2}\right) \times \underset{x\to 0}{\text{Lt}}\frac{\frac{\text{Si ndx}}{\frac{dx}{2}}}{} \times 2}{\sqrt{\underset{dS\to 0}{\text{Lt}}\text{Sik}(x+dx)} + \sqrt{\text{Sik } x}}$$

$$\therefore \frac{dy}{dS} = \frac{\text{Cos } S}{\sqrt{\text{Sin } S} + \sqrt{\text{Sin } S}} = \frac{\text{Cos } S}{2\sqrt{\text{Sin } S}} \text{ Ans.}$$

i.. Differentiate $\text{Sin}^2 x$ w.r.t. x from the 1st Principle.

Let $y = \text{Sin}^2 x$ ----------- (1)

or, $y + dy = \text{Sin}^2 (x + dx)$ ---------- (2)

or, $dy = \text{Sin}^2 (x+dx) - \text{Sin}^2 x = \text{Sin}(x+dx+x)\,\text{Sin}(x+dx+x)$

$= \text{Sin}\left(2x + \frac{dx}{2}\right) \text{Sin } dx$

or, $\underset{x\to 0}{\text{Lt}} \frac{dy}{dS} = \underset{x\to 0}{\text{Lt}} \text{ Sin}\left(2x + \frac{dx}{2}\right) \times \underset{x\to 0}{\text{Lt}} \frac{\text{Sin } dx}{dx}$

$\therefore \frac{dy}{dS} = \text{Sin } 2x$ Ans.

8. $\sqrt{x}$

9. x^3

Differentiation of constant, sum, product and quotient of two functions:

Differentiation of constant quantity:

Let $y = c$ -----------(1)

$y + dy = c$

$dy = 0 \rightarrow \underset{x\to 0}{\text{Lt}} \frac{dy}{dS} = 0 \therefore \frac{dy}{dS} = 0$

i.e. Differentiation of constant quantity is zero

Differentiation of sum of two functions:

Let $y = u + v$---------- (1)

$y + dy = (u + du) + (v + dv)$-------- (2)

or, $dy = u + du + v + dv - u - v = du + dv$

or, $\underset{x\to 0}{Lt}\frac{dy}{dS} = \underset{x\to 0}{Lt}\frac{du}{dS} + \underset{x\to 0}{Lt}\frac{dv}{dS}$

∖ $\boxed{\frac{dy}{dS} = \frac{du}{dS} + \frac{dv}{dS}}$ → It is also true for more than two variables.

Differentiation Coefficient of a sum of a functions

Let $y = u+v+w + \ldots\ldots$--------- (1)

Where u, v,w .... are the function of x

or, $y + dy = (u + du) + (v + dv) + (w + dw) + \ldots\ldots$ ------ (2)

Subtract eq^n (1) from eq^n (2) we get.

$dy = du + dv + dw + \ldots\ldots$

or, $\frac{dy}{dS} = \frac{du}{dS} + \frac{dv}{dS} + \frac{dw}{dS} + \ldots\ldots$

or, $\underset{dx\to 0}{Lt}\frac{dy}{dS} = \underset{dx\to 0}{Lt}\frac{du}{dS} + \underset{dx\to 0}{Lt}\frac{dv}{dS} + \underset{dx\to 0}{Lt}\frac{dw}{dS} + \ldots\ldots$

or, $\frac{dy}{dS} = \frac{du}{dS} + \frac{dv}{dS} + \frac{dw}{dS} + \ldots\ldots$

i.e. d.c. of y = d.c. of u + d.c. of v + d.c. of w + ...

Differentiation of Product of two functions:

Let $y = u\,v$ --------- (1)

$y + dy = (u + du)(v + dv)$ ------- (2)

or, $dy = (u + du)(v + dv) - uv = uv + udv + v\,du + du\,dv - uv$

or, $\underset{dx\to 0}{Lt}\frac{dy}{dS} = u\underset{x\to 0}{Lt}\frac{du}{dv} + \underset{x\to 0}{Lt}\frac{du\,dv}{dS}$

As du and dv are very small quantities their product will be also very small so it can be neglected i.e. Product of two functions = 1st func. × d.c. of 2nd func. + 2nd func. × d.c. of 1st function.

∖ $\boxed{\frac{dy}{dS} = u\frac{dv}{dS} + v\frac{du}{dS}}$

Differentiation of quotient two functions:

Let $y = \frac{u}{v}$ --------- (1)

$y + dy = \frac{u + du}{v + dv}$ ------- (2)

\ $dy = \frac{u + du}{v + dv} - \frac{u}{v}$

$= \frac{uv + v\,du - uv - u\,dv}{v\,(v + dv)}$ As $dx \to 0$, $dv \to 0$ \ $v\,(v + dv) = dv$

or, $\underset{dx\to0}{Lt} \frac{dy}{dS} = \frac{v \underset{dx\to0}{Lt} \frac{du}{dx} - u \underset{dx\to0}{Lt} \frac{dv}{dx}}{v^2}$ \ $\frac{dy}{dS} = \frac{v\frac{dy}{dx} - u\frac{dv}{dx}}{v^2}$

i. e. D. c. of quot i entof t wofunct i ons= $\frac{(\text{Den.}) \times \text{d. c. of num.} - \text{num.} \times \text{d. c. of Demo}}{(\text{Demo})^2}$

Example: 1. Find d.c.of $x^{4/3} + 4\sec x - 3\log x + 5e^x$

\ $\frac{dy}{dS} = \frac{d\,(S)^{4/3}}{dS} + 4\frac{d(\operatorname{Sec} S)}{dS} - 3\frac{d(\log x)}{dS} + \frac{5d\,(e^x)}{dS}$

$= \frac{4}{3} x^{1/3} + 4 \operatorname{Sec} x \tan x - \frac{3}{x} + 5e^x$ Ans.

2. Find the d.c. of $x^2 \operatorname{Sin} x$

Let $y = x^2 \operatorname{Sin} x$

\ $\frac{dy}{dS} = x^2 \frac{d(\operatorname{Sin} S)}{dS} + \frac{\operatorname{Sin} d\,(x)^2}{dS} = x^2 \cos x + 2x \operatorname{Sin} x$ Ans.

3. Find d.c. of $\frac{x^2}{\operatorname{Sin} x}$

$\frac{dy}{dS} = \frac{\frac{\operatorname{Sin} d\,(x)^2}{dx} - \frac{x^2 d\,(\operatorname{Sin} x)}{dx}}{(\operatorname{Sin} x)^2}$

or, $\frac{dy}{dS} = \frac{2x \operatorname{Sin} x - x^2 \cos S}{\operatorname{Sin}^2 S}$ Ans.

Differentiation of inverse trigonometrical functions:

1. Find d.c. of $\sin^{-1} x$
   Let $y = \sin^{-1} x$

$\backslash$ x = sin y

or, $\frac{dS}{dy} = \cos y \ \backslash \ \frac{dy}{dS} = \frac{1}{\text{Cos}\, y}$

or, $\frac{dy}{dS} = \frac{1}{\sqrt{1-\text{Sin}^2 y}} = \frac{1}{\sqrt{1-x^2}}$

2. $y = \cos^{-1}x \setminus \frac{dy}{dS} = -\frac{1}{\sqrt{1-x^2}}$

3. $y = \tan^{-1}x$

$\setminus x = \tan y$

$\frac{dS}{dy} = \sec^2 y \setminus \frac{dy}{dS} = \frac{1}{Sec^2 y}$ | $1 + \tan^2 q = Sec^2 q$

$\setminus \frac{dy}{dS} = \frac{1}{1+\tan^2 y} = \frac{1}{1+S^2}$ Ans.

4. $\cot^{-1} x = -\frac{1}{1+S^2}$

5. $\sec^{-1}x = \frac{1}{S\sqrt{x^2-1}}$

6. $cosec^{-1} x = -\frac{1}{S\sqrt{x^2-1}}$

Differentiation of Function of a Function:

Theorem: -To find the differential coefficient of a function of a function.

If y be a function of u and u be a function of x then $\frac{dy}{dS} = \frac{dy}{du} \times \frac{du}{dS}$

Let dx, du and dy are the corresponding increment in x, u and y respectively

Hence when dx →0 then du → 0 also

We know from elementary algebra

$\frac{dy}{dS} = \frac{dy}{du} \times \frac{du}{dS}$ Taking limit as dx →0 and du →0 also.

$$\underset{dx\to 0}{Lt} \frac{dy}{dS} = \underset{du\to 0}{Lt} \frac{dy}{du} \times \underset{dx\to 0}{Lt} \frac{du}{dS} \setminus$$

$$\boxed{\frac{dy}{dS} = \frac{dy}{du} \times \frac{du}{dS}}$$

Let y be a function of u, u be v and v be function of x then

$$\boxed{\frac{dy}{dS} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dS}}$$

Example- 1. To find the d.c. of $\cos\sqrt{x+3}$

Let $y = \cos\sqrt{x+3}$ Let $u = \sqrt{x+3}$

$\setminus \frac{dy}{dS} = \frac{d[\cos\sqrt{S+3}]}{d(\sqrt{S+3})} ´ \frac{d(\sqrt{S+3})}{d(S+3)} ´ \frac{d(S+3)}{dS} = -\sin\sqrt{x+3} ´ \frac{1}{2\sqrt{S+3}} ´ 1$

$= -\frac{1}{2\sqrt{S+3}} \sin\sqrt{x+3}$

2. $\sin\{\cos(\tan\sqrt{x})\}$ 3. $Y = x^2 \cos(\log x)$ 4. $\log\frac{1-S^2}{1+S^2}$ 5. $e^{\sqrt{\cot S}}$

Explit Function: The function in which the relationship between x and y can be expressed directly eg. y= sin x ,

$y= 5x^2 + 3x +1$

Implicit Function: The function in which the relationship between x and y can not be expressed directly. eg. xy = sin(x+y)

Example: If $x^2 + y^2$ = Sin (xy) then find the value of $\frac{dy}{dS}$

or, $2x + \frac{2y\,dy}{dS} = \cos(xy)\,[y + x\frac{dy}{dS}]$

$= y\cos(xy) + x\cos(xy)\frac{dy}{dS}$

or, $\frac{dy}{dS}\,[x\cos(xy) - 2y] = 2x - y$

$\backslash\ \frac{dy}{dS} = \frac{2x - y\ \cos(Sy)}{x\,\mathrm{Cos}(xy) - xy}$

Find d.c. of the following if:

(i) $x^2 y^2 = \tan(x^2 + y^2)$ (ii) xy = Sin(x+y)

(iii) $x^a y^b = (x - y)^{a+b}$ (iv) $\log xy = x^2 y^2$

3. Find $\frac{dy}{dS}$ when $\log xy = x^2 + y^2$

Parametric Function: Sometimes the relationship between two variable x and y is represented by third terms and that term is known as parameter.

Example: x= a cosq, y = b sinq

$$\frac{dy}{dS} = \frac{dy}{dq} \times \frac{dq}{dS} = \frac{dy}{dq} / \frac{dq}{dS}$$

Example: Find $\frac{dy}{dS}$ when x = a (q + sinq), y = a (1- cosq)

$\frac{dS}{dq} = a\,[1 + \cos q]\ \backslash\ \frac{dq}{dS} = \frac{1}{a[1+\mathrm{Cos}q]}$ and $\frac{dy}{dq} = a[0 + \mathrm{Sin}\,q]\ \backslash\ \frac{dy}{dS} = \frac{dy}{dq} \times \frac{dq}{dS} = \frac{a\,\mathrm{Sin}q}{a[1+\mathrm{Cos}q]}$

$= \frac{2\,\mathrm{Sin}\frac{q}{2}.\ \mathrm{Cos}q/2}{2\cos^2 q/2} = \tan\frac{q}{2}$ $\qquad \cos 2q = 2\cos^2 q - 1$

Example: Find $\frac{dy}{dS}$ when x = a(q - sinq) and y = a (1 − cos$q$)

Differentiation of one variable w.r.t. other independent variable:

1. Differentiate x^7 w.r.t. x^4

Let y= x^7 and z = x^4

we have to find out $\frac{dy}{dZ}$

$\frac{dy}{dS} = 7x^6$ & $\frac{dZ}{dS} = 4x^3$

$\setminus \frac{dy}{dZ} = \frac{dy}{dS} ´ \frac{dS}{dZ} = \frac{7S^6}{4S^3} = \frac{7}{4}x^3$ Ans.

2. Differentiate tan x with w.r.t. x^2

3. Differentiate secx w.r.t. tanx

4. Differentiate $\frac{x}{\text{Sin } x}$ w.r.t. sinx

5. Differentiate log x^2 w.r.t. e^x